New (and old) reflections on some misunderstandings in the sources and in the reception history of Die Kunst der Fuge
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It seems, unfortunately, highly unlikely that today, more than two hundred and fifty years after the putting on sale of the engraved plates of the so-called Kunst der Fuge by Carl Philipp Emanuel1, one could still find something of that material or of some other primary and unknown sources, containing perhaps the Abklatschvorlage, almost entirely missing, of the music released in the two editions, of 1751 and 1752. Not even Wilhelm Rust could in fact trace even the small sheet of paper (“ein kleines...Zettelchen”) on which C. P. E. Bach noted down by himself: “Herr Hartmann hat das eigentliche”2.

Things being as they are, an impressive mass of literature has so far tried, using disparate methodological criteria and starting from often incompatible cultural contexts and interests, to deal with the complexity of the problems raised by this masterpiece, finishing frequently by amassing further misunderstandings of various nature. These, in turn, perpetuated by the subsequent scientific researches, have drawn always more interpreters and scholars away from a critical and direct examination of some clear Bach’s intents, which can be objectively deduced from the autograph sources and from the in-depth knowledge of the early composition techniques, which constitute the basis of the admirable language peculiar to this musical cycle. Unfortunately some serious misunderstandings date back to those who could have had Bach’s materials at hand, therefore they originate paradoxically with the printed edition itself, in that final phase of the engraving process which took place short after the Kantor’s death.

Several inconsistencies and evident mistakes have been found from the 19th century on: the extensive methodological, cultural, scientific and musicological approach of such figures as Hauptmann, Rust or Spitta was supported also by an in-depth knowledge of the practical music, and therefore of the composition.

In the following lines, far from tackling the problems with a spirit of the one who believes, barricaded in the reign of certitude, to be able to succeed in finding the definitive answer, the present study is the result of an in-depth research on the primary sources and on the specialized literature concerning the Kunst der Fuge undertaken long since, which has resulted in my recording of the work on different harpsichords (among them a Grosses Clavicymbel in Thuringian style with a 16’ stop), which will be published by Brilliant Classics in a box containing also the Musikalisches Opfer and the Canonische Veränderungen BWV 769 (the last ones performed on the Trost organ in Waltershausen), together with all the primary sources of the recorded works, attached in a digital format.

I would like to thank Joshua Rifkin who made me aware, during a Bachian “chat” we had, of the new researches by Gregory G. Butler, which will be discussed in the following pages. It is to him that I owe also the intercession with Prof. Butler who, with great courtesy, let me read in advance the not-yet-printed version of his important study published in About Bach, 2008.

1 Bach-Dokumente III, 683.
2 Christoph Wolff – The Last Fugue: Unfinished? in Current Musicology 19 (1975), p. 77 – proposes the identification of “Herr Hartmann” with the Silesian Friedrich Traugott Hartmann (1749-1833), in touch with Marpurg and active as a publisher in Elbting in the East Prussia. Such a hypothesis was not mentioned by Wilhelm Rust (in his foreword to the BGA edition 25/1,1875, p. XIX), who reported instead the identification proposed by the formerly librarian of the Royal Library in Berlin S. W. Dehn (Caecilia 24, 1845, p. 22) with the Rathsmusikus Johann Samuel Hartmann, in a friendly relation with C. Ph. E. Bach in Hamburg, or with Johann Hartmann, from 1768 Konzertmeister in Copenhagen. Klaus Hofmann in KB of the NBA (p. 60) brings to the discussion on the identification also the names of the Hamburgian musicians Johann Georg Hartmann (1754-1784) and Hinrich August Ferdinand Hartmann (1770-1838).
some of the old and new misunderstandings will be reviewed, in the hope of reopening a
debate around a series of critical issues, somehow left aside in the last decades.

I

About the *Fuga a 3 soggetti* BWV 1080/19 anew

Recently an important contribution by Gregory Butler\(^3\) has appeared, reopening with new
and convincing argumentations the issue of the *Fuga a 3 soggetti* and getting (in opposition
to his previous article of 1983) to consider that piece extraneous to the KdF and, not least
importantly, as a composition designed to elaborate three and not four subjects. Therefore
Butler arrives through new paths to the 19th century Hauptmann’s assumption, which
will become later the one of Rust and Spitta, almost forgotten by interpreters and scholars
from the 20th century on, except for Gustav Leonhardt\(^4\). I have always been convinced
both of the extraneousness of that wonderful fragment to the KdF, as well as of the
absurdity of the Nottebohm’s sensational “discovery”\(^5\), which evidently didn’t consider
the most elementary rules of composition and overlooked the indisputable necessity of the
intervallic, rhythmical and rhetoric contrast of the subjects and countersubjects in the
fugue composition.

Some considerations will be therefore proposed on the difficult (and, standing to the actual
state of the sources in our possession, unsolvable) issue of the fragmentary fugue, starting
from the many assumptions present in the above mentioned work by Butler, shared also
by me\(^6\).

The aforementioned “discovery” has until now been attributed to Nottebohm with his
article of 1881\(^7\): the writer has found, however, an English analysis, preceding even the
*Bach Gesellschaft* edition cured by Rust, where Higgs\(^8\), having proposed the eventuality of


\(^4\) See the notes on his recording for Deutsche Harmonia Mundi of 1969, where BWV 1080/19 is not performed.

\(^5\) I have presented a report questioning the affiliation of the *Fuga a 3 soggetti* to the cycle of contrapuntal variations on
the main subject of the KdF at the seminar *Bach – per una memoria costante* which took place at the Venice Music
Conservatory in May 2001.

\(^6\) Butler2008 p. 116-117: „One of the most persistent myths surrounding the Art of Fugue is that the collection was to be
crowned with a quadruple fugue, the “incorrectly” titled *Fuga a 3 Soggetti* that survives in autograph form. Never mind
that C. P. E. Bach in his advertisement for the second edition includes the work among the “last pieces” along with the
version for two keyboards of Contrapunctus 13 and the chorale “Wenn wir in höchsten Nöten sein” after his discussion
of the “manifold fugues ... composed upon one and the same principal theme.” Never mind that the watermark of the
paper and Bach’s hand in the autograph score of the *Fuga a 3 Soggetti* date the manuscript to the very end of the
composer’s life and thus separate this source in time from the composition of the other works in the collection. Never
mind that the principal subject of the Art of Fugue is uncomfortably close in its formulation to the first subject and its
supposed combination with the three subjects of this fugue is far from convincing, contrapuntally. Never mind that
concluding the collection with a canonic appendix and not a fugue would be much more in keeping with Bach’s *modus
operandi* in the late speculative works. Despite the fact that there is not a shred of evidence that this work belongs in the
collection and much that argues against this assumption, Bach scholars are unwilling to give it up and so the myth
endures. At one time I subscribed to it myself and in my earlier essay on ordering problems, I suggested that this work
would have filled the gap of six pages preceding the concluding canonic complex.”

\(^7\) G. Nottebohm, *J. S. Bach’s letzte Fuge*, in *Musk-Welt* 1880/81, pp. 232-236 and 244-246.

combining the three subjects with the KdF main one, reached then a conclusion that this proposal was absolutely to be rejected.

Proceeding in an orderly manner, with the aim to reconstruct for once all the intrigue of the Fuga a 3 soggetti, we will implement Butler’s reasoning. After Bach’s death, who took his handwritten material to carry out the edition of the print of this keyboard cycle – and it wouldn’t be erroneous to think that more people (and in different places?) were responsible of it, as we will see in the subsequent considerations – had to face the present fragmentary Beilage 3. Or, following the partially debatable proposals by Wolff, anyhow couldn’t trace between the composer’s sheets a continuation (probably in a rough copy?) of the Fugue, which was regarded as incomplete right from the beginning.

The piece would get inserted in the print, together with the Choralvorspiel BWV 668, the earlier version BWV 1080/10a and BWV 1080/18, being amongst the compositions on which Bach had been working in his last months. There is no evidence, neither of Marpurg, nor of the Nekrolog authors, referring to the fact that the Fugue would belong to the strictly intended cycle, and therefore not of the printed volume entitled subsequently Die Kunst der Fuge but of the cycle of contrapuntal variations on the main subject. Neither is there any testimony stating that the fragment would contain or should have contained the main subject of the KdF. It is important to remind here the Avertissement of the 7th of May 1751, which up till now is the earliest document which describes, while announcing it, the contents of the printed volume entitled Die Kunst der Fuge.

After having specified that the work consists of 24 examples (thus the 24 engraved pieces, including the Kirchen-Choral), the author of the article, probably Carl Philipp Emanuel, lingers then over what he defines a “vollständiges Werck” containing fugues of various kind in two, three and four voices, with one, two or more subjects, inverted, in contrary motion, per augmentationem, per diminutionem, at the octave, at the tenth, at the twelfth, canons of all kinds. All these – as it is being remarked – elaborated on only one subject and in the same tonality, D minor. The article describes subsequently the merits and the advantages of having presented the material in open score, among other things delegitimizing all the following and inappropriate dispute around the instrumental destination through a primary importance annotation: “Es ist aber dennoch alles zu gleicher Zeit zum Gebrauch des Claviers und der Orgel ausdrücklich eingerichtet”. Only after several lines, having described the cycle of the counterpoints on the same subject, the last pieces are being mentioned: “Die letzten Stück sind zwey Fugen für zwey unterschiedene Claviere oder Flügel, und eine Fuge mit drey Sätzen, wo der Verfasser bey Anbringung des dritten Satzes seinen Namen Bach ausgeführet hat. Den Beschluss macht ein Anhang von einem veirstimmig ausgearbeiteten Kirchen-Choral [...]”.

These last pieces are deliberately described aside (and later) compared with the fugues on the same subject. Therefore, also for those responsible of the edition of the cycle, or at least for the author of the Avertissement, BWV 1080/19 (besides BWV 668 and, as will be shown later, BWV 1080/18) constituted something different, not homogenous with the cycle on the main subject. Furthermore, regarding the Fuga a 3 soggetti, an explicit reference is being

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made only to the third subject, the one on B-A-C-H. The Kunst der Fuge – which, as a printed volume as it appears today, became, starting from the title, an editorial product of Carl Philipp Emanuel and Marpurg (in any case of the editors), rather divergent from the original one conceived by the composer – was meant to offer to its buyer a didactic corpus on the fugue of the recently deceased Sebastian Bach, renowned in the musical circles as Fugenmeister: inclusion of all what was available on the death of the author, seemed as the best solution to the editors to give glory to the memory of the Kantor and to render the expensive volume as desirable as possible to its potential buyer. Even the publishing of the previous version of the Contrapunctus 10 BWV 1080/10a (significantly entitled, probably after Bach’s death, Contrapunctus, possibly to indicate clearly it was an elaboration of the main subject of the variations’ cycle) might be interpreted from the viewpoint of the willingness to print as much material as possible, and not as a simple oversight of the editors who didn’t realize that the piece has been re-elaborated, deeply modified and improved by Bach in the tenth fugue already engraved on plate; even more if we think again that in the “24 Exempel” also the Kirchen-Choral has been included, undoubtedly a contrapuntal piece, but not in D minor, not in a fugue form and not on the main subject. These resulted in publishing a heterogeneous volume, certainly distant from the project of a cycle of variations on the same subject to be printed as a practical treatise on fugue, with no theoretical explanations, hence much more useful as a practical applied model for a counterpoint scholar and, at the same time, keyboard virtuoso.

Furthermore, it seems likely that who inserted those “heterodox” pieces and was in charge of the final engraving phase, was absolutely not aware in detail of the intents of the deceased Kantor. It’s probable that there were some vague and not exhaustive rumors going around that Bach would have wanted to compose one more piece on the KdF subject to complete the cyclical structure, in accordance with his last plans. The discrepancy of the first-hand testimonies is clear if we confront the contents of the two passages which can be found in Marpurg’s Vorbericht11 and in the Nekrolog12:

Marpurg, 1752
Er wurde von demselben [Tod] mitten unter der Ausarbeitung seiner letzten Fuge, wo er sich bey Anbringung des dritten Satzes nahmentlich zu erkennen giebet, überrascht.

Nekrolog, 1754
Seine letzte Krankheit, hat ihn verhindert, seinem Entwurfe nach, die vorletzte Fuge völlig zu Ende zu bringen, und die letzte, welche 4 Themata enthalten, und nachgehends in allen 4 Stimmen Note für Note umgekehrt werden sollte, auszuarbeiten.

Two essential facts should not be neglected. The Beilage 3, at least until its penultimate page, is a fair copy (with some corrections added) in the form of Abklatschvorlage, prepared purposely to be transferred directly on the plate, hence written only on the recto with the verso left blank. Moreover, it is in form of Klaviernotation and not in open score like all the remaining pieces of the cycle.

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11 Dok III, 648
12 Dok III, 666, p. 86
The setting diversity must have been noticed also by the editors who had to prepare an Abklatschvorlage deliberately in open score for their printed edition, for the evident reasons of the notational coherence. Butler\textsuperscript{13} observes that the alterity of the piece was already clear for who inserted it, so much so that the term “Contrapunctus” was not used and the fugue was collocated in the final position, as an appendix, together with BWV 1080/18 (also called “Fuga” and not “Contrapunctus”, unlike BWV 1080/10a) and BWV 668. The status of Abklatschvorlage indicates precisely Bach’s intent to publish the piece in Klaviernotation: Butler\textsuperscript{14} believes that this wonderful keyboard piece should have constituted Kantor’s annual contribution to be inserted in the Pacquet circulating among the members of the Mizler society, which would be much more suitable for this scope than a voluminous and expensive edition of the keyboard variations cycle which he was working on. Moreover, the signature expressed by the third subject seems to fall within the idea of a musical farewell for the members before his retirement when he turned 65.

He who thinks it’s possible to overlap the principal subject of the KdF with three subjects of BWV 1080/19 does so \textit{in primii} to justify the affiliation of the piece to the variation cycle, \textit{in secundiis} to follow the indication which can be found in the Nekrolog. The indication, however, mentions a presumed mirror fugue on four subjects, still to be composed. It seems furthermore unlikely that Bach would be really thinking on such a piece. Butler sets the reasoning of the Nekrolog as follows: the penultimate, unfinished fugue is BWV 1080/13.2 whose Abklatschvorlage was not finished (or not even initiated) by Bach, as it appears clear from the prearrangement of the bars all of the same width at the page 42 of the print compared with \textit{versio recta} BWV 1080/13.1\textsuperscript{15} (the issue will be treated in detail in the next chapter); the last piece, a possible Contrapunctus 14, should have been a third mirror fugue in four voices (and not four subjects, which would be absolutely unlikely from the contrapuntal point of view), which would form a subgroup of three \textit{Contrapuncti inversi} symmetrically in 4, 3 and 4 voices. The “4 Themata” could be fruit of umpteenth imprecise news, misunderstood or even swollen, as we will see below with regard to the trustworthiness of the passage of the Nekrolog). However matters stand, once it has been overlapped, to be consistent with the Nekrolog, the supposed fourth subject (the main one of the KdF), one would never obtain a reversible fugue. If one would like to revert in accordance with the procedure which we will see in the next chapter the material of the fragment (as I have tried to do) we would get a musical nonsense starting from the first bars. Why then insist on the quadruplicity of the subjects (according to the sole Nekrolog) and omit the invertibility (always testified only by the Nekrolog), when eventually both proceedings should take place simultaneously and, always according to the biographers, in a fugue that has never been written?

Marpurg doesn’t mention at all the quadruplicity of the subjects in his foreword. Hence in this case it would result that, if no fourth subject was missing, according to Marpurg the fugue should be on three subjects. Where is then the main subject of the KdF? Hofmann suspects that the paternity of the problematic passage of the Nekrolog should be attributed to Mizler and not the main authors C. P. E. Bach and Agricola (even if Carl Philipp

\textsuperscript{13} Butler2008, p. 116-120.
\textsuperscript{14} idem
\textsuperscript{15} idem, p. 119
Emanuel wrote later that Mizler was responsible only for the brief part of text concerning the *Societät*. In the *Nekrolog* the KdF is being commented amongst the printed works and even with the verb in its *Perfekt* tense form: “Dieses Werk ist erst nach des seeligen Verfassers Tode ans Licht getreten”. We know that the text was at disposal of the physician and former student of Bach already in March of the 1751 when the KdF almost certainly was not printed yet. Therefore Hofmann wonders whether the debated passage couldn’t constitute a subsequent and unreliable addition by Mizler, maybe of 1754, the year when he published the *Nekrolog*.

The overlapping of the main subject as the fourth subject is preposterous for many reasons. Bach has always followed the rule of *varietas* in choosing the subjects of all his contrapuntal polythematic compositions. Such a rule is a *conditio sine qua non* to compose a fugue on more subjects. The first subject, with its initial leap of an ascending Fifth (D-A) and descending motion towards the Mediant (F), would have never combine with a subject with pretty much the same beginning and with the rhythmical proceeding too much alike. That was again well understood by Higgs who wrote: “May not this fugue have found its way into *The Art of Fugue* by accident? [...] The want of connection shown by the absence of the original subject, and also the similarity of the first subject with the original subject, does not agree with Bach’s habit of selecting subjects for combination of marked contrast”. Such a resemblance would already by itself exclude the legitimacy of any further attempt to overlap the main subject as the fourth one. The first subject of this wonderful fragment is in fact in the *Stylus antiquus*, by way of *Ricercare grave*, the second one takes the form of an animated diminished melody in the violin style and almost like a *canzona*, and the third one is solemnly dramatic and by *genus chromaticum*. The contrast – which, according to the implicit rules of the fugue composition, needs to be melodic (by step/by leap, ascending/descending motion, diatonic/chromatic), rhythmical (*stylus gravis, stylum diminuitus*) and harmonic – is admirable. Furthermore, the composition keyboard style of the monumental piece is very different from the one of the *Contrapuncti* of the KdF: the first section is somewhat reminiscent of the movements and the stylistic elements of the *Ricercar a 6* of the *Musikalisches Opfer*. All leads to think, as it has been said, of a final contribution for Mitzler (to be engraved on two staves), an erudite *summa* of all keyboard styles, from Frescobaldi to the more modern ones, with an ingenious and dramatic melodic signature of a supreme harmonic proceedings.

The lack of contrast between the first subject of BWV 1080/19 and the main subject of the KdF led Bitch and Chailley to challenge Nottebohm (who revealed with his pseudo-discovery – a poor knowledge of the historical and Bach’s counterpoint) and to become convinced that such a first subject would be a part of the main subject of the KdF, hence it

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16 *Dok III, 803*
17 *Dok III 637*
18 KB della NBA, p. 96.
19 Doubting the accuracy and the veracity of the testimonies in the *Nekrolog*, and referring only to the Marpurg’s one, which talks about the *Fuga a 3 soggetti* without explicitly considering it as belonging to the variations cycle on the main subject in a strict sense, it would be even legitimate to bring into question the incompleteness itself of a work, maybe already engraved on plates, almost entirely under control of Bach himself before his final illness. Neither Spitta, nor Rust, considered in fact the KdF as incomplete.
itself a main subject. And as a consequence BWV 1080/19, of course in this sense, a Fugue on only three – and not four – subjects. This idea has already found expression – one more time in the British land and well before the quoted French musicologists who don’t seem to have been acquainted with it – by a German-born Augustus Frederic Christopher Kollmann who wrote in 1799: “The first Subject [...] is part of that on which he has composed all the twenty-three inimitable Fugues, and Canons in the form of Fugues, of the said most estimable work”. However, also this is unsustainable, lacking in this first subject of the Fuga a 3 soggetti all what makes distinctive and so malleably unique the main subject of the KdF, above all the diminished Seventh in the middle of the melody, so dear to Bach, implicit in the C sharp (which, through an inversion by a contrary motion turns into B flat). A subject in d minor in the Ricercare style, presents very often an initial leap of the Fifth that descends to the Mediant. This does not relate it in any way to the KdF subject, which after having touched Tonic-Dominant-Mediant returns straightaway to the Tonic and continues in a quite different, peculiar manner. One could be even tempted to think that, as mentioned, if the fugue wouldn’t be in D minor or wouldn’t get inserted in the print by the editors, no one would have built so many castles in the air and created numerous suites of misunderstandings.

Furthermore, from the technical point of view, the presumed final overlapping of the fourth subject by way of final stretto doesn’t make by itself a quadruple Fugue, as it is commonly defined today: in the strict sense of the word, the Fuga a 3 soggetti is, following Mattheson’s terminology of the 23rd chapter of the Vollkommener Capellmeister, a double Fugue on three subjects. In order to be quadruple, the four subjects, in relation of Quadruple Counterpoint (probably at the Octave), should be able to be combined in 24 (1x2x3x4) possibilities. The combination proposed by Nottebohm, moreover full of not convincing forcings and rhythmical modifications, doesn’t work for the most part of its possible reversions, as it has been already witnessed by Higgs with reference to his analog and former experiments. Never would Bach put that Fifth in Double or Quadruple Counterpoint, which reversed would bring forth a dissonant and not justified Fourth (see Fig. 1). Moreover, the two incorrect, quasi-parallel Fifths between Soprano and Alto in bars 3-4 of the Fig. 1 sound badly in this unconvincing combination. In Nottebohm the subject of the KdF can appear only in an upper part with the bass exposing the first or the third subject. (According to Wolff, moreover, most of the combinations of the presumed four subjects are not playable on the keyboard).

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22 Dok III, 1021, p. 575.
23 Also Hawinks in Dok III, 817 defines BWV 1080/19 “a double fugue in three subjects, in one of which he introduces his name”. Hawkins knew about the piece from Johann Christian and didn’t mention its belonging to the KdF.
Finally, some considerations about the so-called fragment X\textsuperscript{25}: if it is true that, when composing a polythematic fugue, one prepares first a simple draft putting in the score one on another the various contrasting subjects in order to make them agree in conformity with the rules of the multiple counterpoint, then it is equally unlikely that Bach (nor any other experienced contrapuntist) would have started composing at first the whole final section. A final section, where all the subjects interlace, needs in fact to make use of the greatest number of the subject elements elaborated up to that moment; what’s more, it’s necessary to arrive there with a tension created section by section. Who improvises a fugue (Bach very likely followed a composition approach which originated from the innate practice of improvisation, hence putting on paper a “fingers-born” idea in the temporal sequence) starts always from the beginning, after having analyzed the possibilities of the eventual countersubjects or other subject to elaborate. Nor it is possible to share the idea, which Wolff follows only to stick with coherence to the \textit{Nekrolog}, of the section with the contemporaneous elaboration of the four subjects, followed in turn by another section with the contrary motion inversion and the reversion (as we will see in the next chapter) only of that previous section inside the very same piece. As far as the author knows, no such pieces, perilously loaded with disproportion, exist neither in Bach, nor in all the literature.

The structure of the \textit{Fuga a 3 soggetti} is by itself well outlined and very long: after the elaboration of the first subject in \textit{Ricercare}-like style (which is used also by contrary motion), it passes to a fugue on the second subject of \textit{Canzona}, then combined in Double Counterpoint with the first subject. What follows is a section by way of Frescobaldian \textit{Adasio} of “Durezze”, a proper fugue only on the third subject, in the end of which starts a wonderful overlapping of the three subjects which suddenly interrupts. Whether it has been finished in some rough copy not found anymore neither by the editors or it has never been composed, will remain perhaps forever an unsolved issue\textsuperscript{26}.


\textsuperscript{26} Moreover, from a contrapuntal point of view, it is unlikely that in a section where the three subjects overlap, a fourth one appears, especially if not previously elaborated in the course of the piece. On the other hand, it is also unthinkable that another section elaborating the subject of the KdF would follow the interrupted one (with the stretto of the three subjects), and even a sixth section with all the four subjects.
Another musical misunderstanding around the structure of the cycle, laden with critical consequences on the history of its reception and, more in general, on the understanding of the masterpiece as a whole, is the one regarding the couple of mirror counterpoints and the Fuga 2 Clav. Its implications have induced many scholars to omit the comparing of the objective compositional issues with the documentary ones and with the sources, presenting in different editions of the work (however accurately edited) some evident mistakes regarding the identification of the matrix of the pieces and their respective inversions. The matter has been discussed many times in the past. Some considerations will follow, fruit of the cycle study approach of a musicus practicus, which agree with various objective data from the sources we possess.

It will be shown that not only the two versions of the Cp XII BWV 1080/12 but also the following couple of pieces BWV 1080/13, which will be called Cp XIII from now on, are in a wrong order in the print compared with the composition process undertaken by Bach and therefore clearly in contrast with his intents of the final ordering of the cycle to be printed. Whereas the scholars agree unanimously for the first case, from Rust on, on considering the piece, which in Schmieder’s catalogue is numbered BWV 1080/12.1, as versio recta of the fugue and the BWV 1080/12.2 as its inversion+reversion, in the case of the Cp XIII the accuracy of the order proposed by the autograph of the early ’40s, restored correctly by Rust (which is reflected in the Schmieder’s numeration), has been put into discussion again by recent critical editions, therefore bringing the pieces back in the wrong order of BWV 1080/13.2 and afterwards BWV 1080/13.1, exactly as in the prints of 1751 and 1752. I will show instead that the order attributed by Rust and, consequently, by the first edition of the Schmieder’s catalogue, is the one intended by Bach, correct from the contrapuntal, compositional and documentary point of view. Hence the recta version of the fugue is BWV 1080/13.1 and its rovescio is BWV 1080/13.2.

The problem will be discussed also in relation to the arrangement for two instruments of the Cp XIII BWV 1080/18, which was published instead in the correct order in the printed edition, even if the writer agrees with many scholars who regarded and regard the insertion of this couple as extraneous to Bach’s editorial project.

Who tackles with the historical counterpoint knows well the process with which it is possible to compose a polyphonic piece in imitative style, which subsequently can be inverted and even reversed. The inversion is intended here as the generation from the original matrix of another line in contrary motion. In the contrario motu inversion the inverted melodic intervals remain the same. In the KdF the pivot around which to invert,
and so the melodic symmetry axis - unlike many Renaissance and Baroque treatises which deal with the inversion from the hexachord theory viewpoint - coincides with F.

Fig. 2 illustrates the inversion scheme of the KdF scale, as it is being used by Bach in the cycle. The notes of the first stave become, when inverted, the ones of the lower stave.

**Fig. 2**

![Inversion Scheme](image)

According to this scheme and to the illustrated principles, the main subject of the KdF (Fig. 3A) generates by contrary motion its inversion (Fig. 3B):

**Fig. 3**

![Subject and Inversion](image)

The main subject of the KdF, elaborated in the Cp. I and II, is used in its inverted form as the subject of Cp. III and IV. In the Cp. V it is being proposed simultaneously in the straight and in the inverted form, and the exposition presents the fugal answers in stretto form, therefore the answer enters when the enunciation of the whole subject has not been finished yet. In the following Cp VI and VII, in addition, the use, respectively, of diminution (in French style) and of the diminution combined with augmentation (and, in both, of the answer in stretto form) is present. In the Cp VIII the main subject in its inverted form becomes combined, in accordance with the Triple Counterpoint at the Octave, with two new countersubjects, generated in turn by the inversion by contrary motion of those used afterwards in the Cp XI. Between this pair in Triple Counterpoint, there is another couple of fugues (Cp IX and X), combining the main subject each with a new countersubject, with respect to the rules of the Double Counterpoint, respectively at the Twelfth and at the Tenth. In both of these fugues in the Double Counterpoint, it is the new countersubject to be first elaborated in the exposition. Furthermore, in the course of these four pieces (which form clearly an internal subgroup of the cycle) all the newly proposed countersubjects are in turn elaborated independently and then combined in many typologies and variously reversed, according to the innermost possibilities of the Double and Triple Counterpoint-based composition.

The following pieces, which we will deal with now, are two ingenious fugues, the first one in four and the second one in three voices, each of them, by virtue of an iron set of rules on

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29 Cf. Butler2008, p. 120 as regards the proposal of grouping the structure of the KdF.
the use of the harmonic consonances, melodic (horizontal) intervals of the single parts and
of the preparation/resolution of the dissonances, allows not only to invert the melodies of
the single lines (fuga contraria) but also, at the same time, to reverse the whole voice
structure (fuga riversa o inversa).
This process has been exposed in various treaties prior to Bach and in others of his times.
Gregory G. Butler\textsuperscript{30} maintains that the Vollkommener Capellmeister by Johann Mattheson (in
print in a Leipzig workshop in the late 1738) could have influenced, as a stimulus, the first
gestation, if not the Bach’s initial idea to create a practical work (thus different from the far
more arid theoretical treatments), which would demonstrate all the possibilities of
developing a fugue, therefore elaborating a subject in all its possibilities, in a complete
form, according to the didactic typologies of the contrapuntal elaboration, ending with the
most complex type of fugue, the one Mattheson calles Kreis-Fuge (“sonst Canon genannt”),
with its various devices. In turn, Mattheson (according to his own admission) followed the
first part of the Documenti armonici by Angelo Berardi (published in Bologna in 1687) in his
treatment of the several types of counterpoint, exposed in the third part of his
monumental treatise. Berardi’s work was known by Sebastian’s cousin, Johann Gottfried
Walther, and a copy of it was almost certainly present in the private library of the Kantor\textsuperscript{31}.
In Berardi’s “Documento XXVIII” from the page 63 on, the four-voices motet Si Deus pro
nobis quis contra nos? by Marco Scacchi is being reported and commented, in its original
version and later in the reversed one (“Risolutione con voltare il Libro al contrario”). The
piece is structured in the same way as Cp XII is: the lines are inverted by contrary motion,
and contemporaneously Soprano becomes Bass, Alto Tenor, Tenor Alto and Bass Soprano.
It is not unlikely that this piece was known by Christoph Berhard (who, through his
teacher Sagittario, has undoubtedly had the opportunity to study the music of the Warsaw
Chapel Master): he has devoted himself to the composition of similar experiments in strict
style, published in 1669 in the Prudentia prudentiana, where the inversions of the pieces are
indicated with the curious name of Revolutiones. This short work, in turn, was certainly a
stimulus for the two admirable Contrapuncti on Mit Fried und Freud by Diderik
Buxtehude\textsuperscript{32}. Both the first and the second Contrapuncti are at first being presented in their
matrix form and then in various evolutiones: the second evolutio of the last Contrapunctus is
modeled exactly according to the principle and the scheme used by Bach in his Cp XII. It
has already been observed that the term “Contrapunctus” used in the print of the KdF,
could have been an influence of this Buxtehuvian work\textsuperscript{33}. Bach undoubtedly knew the
latter through Walther (who discussed it in detail in his Praecepta der musikalischen
Composition finished in 1708, the year of the second transfer to Weimar of his cousin
Sebastian\textsuperscript{34}). Walther, on the other hand, has confronted himself with such a complex
system of multiple and reversible counterpoint devices: in the third verse of the organ

\textsuperscript{30} Gregory G. Butler, Der vollkommene Capellmeister as a stimulus to J. S. Bach’s late fugal writing, in NewMatthesonStudies, 1983, pp. 293-305
\textsuperscript{33} Butler, Der vollkommene Capellmeister as a stimulus to J. S. Bach’s late fugal writing, in New Mattheson Studies, 1983, p. 303.
\textsuperscript{34} D. Yerslay, Bach and the Meanings of the Counterpoint, 2002, p. 13 and foll.
elaboration of the Lied *Herr Jesu Christ, wahr’ Mensch und Gott* a matrix with its contrary and reversed *evolutio* is presented, in accordance with the same scheme of the Cp XII.

What is being meant by reversion, is the inversion (switching) of the structure of the lines entrusted to the different voices: therefore Soprano line becomes the one of Bass, and so on. In the Cp XII, composed in four voices, Bach reverses the whole structure: hence Bass in the *recta* version becomes, at the respective compass, Soprano of the *inversa* version, and so Tenor becomes Alto and Alto Tenor, and finally Soprano Bass.

Composing these Cp XII and XIII, Bach therefore elaborated a matrix generating a subsequent symmetrical reversion, in accordance with two exposed proceedings: the contrary motion (that is inversion) and, by means of Double Counterpoint, the reversion of the voices. These fugues are therefore, following the historical definitions, *Fugae inversae* or, using the term present in the original print, *Contrapuncti inversi*: they use simultaneously the device of the contrary motion and the one of the Multiple Counterpoint (a sort of a Triple Counterpoint in the case of the Cp XIII and a Quadruple Counterpoint in the Cp XII\(^{35}\)). Marpurg talks to this end of "Doppeltverkehrer Contrapunct"\(^{36}\). Mattheson treats those devices (defining them "*al contrario riverso*") in the paragraphs 39-54 of the twenty-first chapter of his *Vollkommener Kapellmeister* (publishing even Bach’s canon BWV 1074 and its two solutions, one in the *recta* form, the other *inversa*, composed in accordance with the same principles we are analyzing here).

In the Autograph of the early ‘40s, with no possibility of any misunderstanding, Bach puts in the notation the *versio recta* of these *Fugae inversae* before and immediately below presents the inversion+reversion. In other words, using a simple metaphor, in a *Contrapunctus inversus* the matrix (*versio recta*) is to the original image as the inversion +reversion (*versio inversa*) is to the image as it appears in the mirror. Obviously the second one derives from the first. What should be therefore noted is that what Bach most likely intended for *Contrapunctus inversus*, following the traditional nomenclature (*Fuga inversa*, *Fuga riversalal rovescio* or *Fuga contraria riversa*), was the entire piece, doubly composed of the matrix and its mirroring, and not the *versio inversa* of the matrix (i.e. the mirroring).

Analyzing the structural aspects of the piece and of its mirror it can be clearly seen that the matrix of the Cp XII (BWV 1080/12.1) presents in the *incipit* of Bass a straight version of the main subject of the KdF (Fig. 5A), to which Tenor gives an answer in A minor and with the necessary initial tonal alteration (Fig. 5C). Such a standard fugal answer complies with

\(^{35}\) As a matter of fact, it is not a Triple or Quadruple Counterpoint in a strict sense, since in the first case the three parts should be combinable in 6 ways (1x2x3), in the second case even in 24 ways (1x2x3x4). In the Cp XII and XIII this does not occur: Bach uses indeed a reversion of the parts, which in turn are melodically inverted in contrary motion. In the Cp XII the intervals between the voices remain unaltered between all the parts, in 1080/12.1 from the bottom to the top, in BWV 1080/12.2 from the top to the bottom, the ascending exposition of the matrix (B/T/A/S) becoming descending in the *rovescio* (S/A/T/B). Furthermore, it is important to note that the use of the Fifth as a contrapuntal consonance allowed in the composition of the matrix does not permit to reverse the lines (original or inverted in the contrary motion ones), according to the schemes of the Double Counterpoint at the Octave, as it would generate dissonant and inadmissible Fourths and Fifths, and - obviously and easily verifiable - falsify all the final cadence. In the Cp XIII the *rovescio* scheme used by Bach 1) modifies, according to the rules of the Double Counterpoint at the Octave, the quality of the consonances in the relation between Soprano 2 and Bass of the matrix, which become Soprano 1 and Soprano 2 in the *rovescio*; 2) leaves unchanged the relation between Soprano 1 and Soprano 2 of the matrix which, becoming Bass and Soprano 1 in the *rovescio*, add only an Octave to the “quality” of the consonances itself; 3) leaves unchanged the relation between Soprano 1 and Bass of the matrix which, becoming Bass and Soprano 2 in the *rovescio*, add only an Octave to the “quality” of the consonances itself.

what was Bach putting in practice in all the previous pieces. Fig. 4 shows the answer, always used by Bach in the expositions of the preceding fugues with the subject in straight form (Fig. 4A) and by contrary motion (Fig. 4B).

Fig. 4

Inverting melodically the material of BWV 1080/12.1, the subject appears in BWV 1080/12.2 in its inverted form as in Fig. 5B.

Fig. 5

Note well, Bach by inverting and reversing BWV 1080/12.2 (therefore in the mirrored version, the versio inversa of the Contrapunctus inversus 12), having bounded himself to maintain the exact melodic structure of the intervals of the matrix, cannot cling to the correct answer that the inverted subject (Fig. 5B) would require, that is the one that can be seen in Fig. 5D. Therefore the fugal answer which enters in Tenor in bar 5 of the versio recta BWV 1080/12.1, once inverted, generates an irregular answer in Alto of BWV 1080/12.2, bar 5: indeed the answer, instead of being in A minor with an initial tonal alteration, appears in G minor, since the newly obtained answer is exactly the inversion (by contrary motion) of Fig. 5C (repeated also in Fig. 6A), as can be seen in Fig. 6B. Since the A minor key is a Fifth up from the Tonic key (D minor), the key becomes, in the inversion of the fugal answer in BWV 1080/12.2, that of a Fifth down, which is precisely G minor (Fig. 6B).

Fig. 6

37 With the exception of the Cp XI, where the initial tonal alteration of the fugal answer does not occur and the initial Fifth leap of the subject is therefore maintained as such in the answer (hence real and not tonal).
This pattern appears only in the two fugue couples that use the “Gegenbewegung” together with the "Verkehrung", to quote Marpurg\textsuperscript{38}, by virtue of the conditions required by the total inversion (hence also of the answer). This aspect becomes then one of the structural elements to understand which one is to be intended as a versio recta, and which one versio inversa. As a matter of fact, all the pieces of the cycle present otherwise, as usual, a standard answer in the Dominant key (Fifth up), that is A minor, as in Fig 4B. Bach has therefore composed first a “regular” fugue - though applying with an unsurpassed mastery the complex obligatory restrictions in order to make everything subsequently invertible by contrary motion and, through the Multiple Counterpoint, reversible in the polyphonic voices - with the exposition of the subject in D minor and its answer in A minor (with regular initial and final alterations). As can be easily - and unanimously - understood, the matrix (the recta version, composed as first) is the Fugue BWV 1080/12.1. The mirror image (what’s more, supplementary and optional) cannot exists without the essential presupposition of the original and primary image, logically written as first and at the top in the Autograph P200.

Such an mistake in the order of the print, as will be shown below, has been also repeated in the so-called Contrapunctus 13: it is indeed undoubtedly presented in a wrong order too in the printed edition. Again the Autograph leaves no doubt: Bach presents there first the versio recta BWV 1080/13.1 and then, below, its mirror image. As has been shown above, we know from the Butler’s important recent analysis (which enhances Koprowski and Wiemer’s\textsuperscript{39} researches) that BWV 1080/13.1 was the only piece of the couple whose Abklatschvorlage was prepared by Bach: he couldn’t instead personally write the Abklatschvorlage of BWV 1080/13.2. Logically, when preparing the Vorlage to be engraved, Bach made a fair copy of the rectus of the piece. It was essential to do this first: once the “generating” fugue was copied in a fair copy, it was not difficult to deduce the “generated” fugue. Such a mirror image, in other words versio inversa of the Contrapunctus inversus 13, was in fact prepared later, as explained also in the previous chapter: Butler points even out the possibility that the Abklatschvorlage of BWV 1080/12.2 could have been edited by the probable engraver Johann Heinrich Schübler\textsuperscript{40}. The wrong order of both versions of the Cp XII as well as of the Cp XIII, has been a serious editorial mistake which must have arisen due to a superficial consideration and lack of careful analysis of the composition techniques. It is not unlikely that all the misunderstanding arose from the wrong interpretation of the probable title of the pieces given by Bach in the Abklatschvorlage of BWV 1080/12.1 and BWV 1080/13.1: if, as can be seen in BWV 1080/13.1, the titles written by Bach were respectively something like “Contrapunctus inversus a 4” or “Contrapunctus inversus a 3”, then it seems that it is the term “inversus” to have caused the misunderstanding of the editors: they have merely seen in it the indication of the versio inversa, misunderstanding the fact that it indicated a genre, in other words a compositional device (the invertibility) of the entire couple of the generating-generated fugues and not the inverted version. It is important to reaffirm that Contrapunctus inversus was for Bach

\textsuperscript{38} Marpurg, op. cit., p. 29 and p. 36.
\textsuperscript{40} Butler2008, p. 112.
the matrix which generated the second symmetric fugue, that is an invertible Contrapunctus, charmingly capable of producing a musical mirroring. Repeating what has been said before, in the historical contrapuntal jargon the Fuga inversa is intended as a fugue invertible by contrary motion and, at the same time, reversible in accordance with the Double, Triple or Quadruple Counterpoint rules. It is not known whether who inverted the natural and logical composition order of the pieces, mislead by a wrong title interpretation, had on his disposal the autograph score of the early ‘40s, which would have clarified the issue. On the other hand, the order in which the two versions of the Fuga a 2 Clav, are presented follows the autograph and the structural logic instead. Here the word “inversus” does not appear, for the mirroring of the fugue is called alius modus. Butler indicates Johann Christoph Friedrich as the possible scribe of the Abklatschvorlage of this keyboard duet elaboration of the Cp XIII\textsuperscript{41}. In accordance with many scholars, I also consider the Fuga a 2 Clav as published in the printed edition not following the Bach’s intent. Indeed it is a domestic and more easily performable version of the original piece in 3 voices (even if a further indispensable copy for the second performer, which the scribe of the Abklatschvorlage of pages 57-60 in the original print might have even used for his copying, is missing) and, more importantly, with the diminished values of the original autograph version and not, as in the print of the Cp XIII, with Stylus antiquus-like doubled values, according to the final will of Bach himself. To this end, it is necessary to remark that who has edited the Abklatschvorlage of BWV 1080/18 had in all likelihood at disposal the Autograph with the top-bottom layout of the two versiones of BWV 1080/13 and/or knew more in-depth the creative process with which Bach composed this music. Indeed, it might have been not clear enough to establish the priorities of the matrix-mirror in the original Beilage 2 layout as it looked like in Bach’s times, if not knowing well this kind of contrapuntal device and/or the Autograph P200. In fact, the Beilage 2 included originally the two versions, without any title, in a single bifolium with each side of the bifolium containing one version\textsuperscript{42}. The musical text of each version was distributed in two pages/columns, respectively on the left and the right of each side of the single big bifolium. In the absence of an unambiguous title at the beginning of the two versiones and/or of the original numbering of the pages (the current numbering was inserted later, in the 19th century), each version was in principle plausible to be played as first: each side of the bifolium contained a version playable from the left to the right with no page turning. The bifolium was then, likely in the 19th century, cut in two single leaves (consequently four sides), in order to insert them in the Mus. Ms. Bach P200 (from which they were later extracted). The two leaves became ordered with a new numbering (20 and 21): the text of BWV 1080/18.2 starts on the page 20\textit{verso} and finishes on the 21\textit{recto}, while the text of BWV 1080/18.1 starts on the 21\textit{verso} and finishes in 20\textit{recto}. The copy of the facsimile I

\textsuperscript{41} Idem.
\textsuperscript{42} See Hofmann, op. cit., p. 22 footnote 12.
have consulted, edited by Hans Gunter Hoke, confuses even more the situation: neither it reproduces the sequence of recto and verso pages as can be seen today in the Beilage 2, i.e. in the unmatched order in which it was forcedly inserted into the Autograph P200 nor reconstructs the original status of the bifolium before it had been cut in two pieces: it presents - in a very arbitrary way - each piece on a separate leaf in recto-verso sequence and not as the two cut handwritten leaves really appear.

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Even more singular is the fact that the copy, consulted by me at the DMS Library of the Bologna University, differs from the one preserved in the Staatsbibliothek in Berlin, which instead presents the mistake of repeating twice the page 20 recto and omitting completely the page 21 recto. Sergio Vartolo, in the recent facsimile edited by him and published in Florence by S.P.E.S. in 2008, on p. 25 of the introduction (which is until now the most complete contribution to the KdF in Italian) mentions his epistolary exchange regarding this issue with Prof. Wollny (Bach-Archiv in Leipzig) and Dr. Brenneis (Staatsbibliothek in Berlin).

To be more clear, on one side the layout of the original bifolium presented the musical text as follows:

<table>
<thead>
<tr>
<th>BWV 1080/18.1 incipit</th>
<th>BWV 1080/18.1 conclusion</th>
</tr>
</thead>
</table>

and on the other side, turning over the bifolium:

<table>
<thead>
<tr>
<th>BWV 1080/18.2 incipit</th>
<th>BWV 1080/18.2 conclusion</th>
</tr>
</thead>
</table>

In perspective, the entire bifolium was so constituted (the two sides are separated by the bold typed line):

<table>
<thead>
<tr>
<th>BWV 1080/18.1 incipit</th>
<th>BWV 1080/18.1 conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>BWV 1080/18.2 conclusion</td>
<td>BWV 1080/18.2 incipit</td>
</tr>
</tbody>
</table>

Later, the bifolium was cut in two independent leaves (therefore following the central vertical line of the chart above) in order to be inserted in the Autograph P200 and hence it was numbered on the verso of each leaf (as it occurs to the remaining numbering added to the whole Ms. P200). The two following leaves present therefore from that moment on the following layout:

<table>
<thead>
<tr>
<th>20 recto</th>
<th>BWV 1080/18.1 conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>BWV 1080/18.1 incipit</td>
<td>BWV 1080/18.2 incipit</td>
</tr>
<tr>
<td>21 verso</td>
<td>BWV 1080/18.1 incipit</td>
</tr>
<tr>
<td>BWV 1080/18.2 conclusion</td>
<td></td>
</tr>
</tbody>
</table>

In the above mentioned facsimile edited by Hoke, owned by the Bologna University, the two published pages of the Beilage 2 are formated without respect to the document, in the following arbitrary way:

<table>
<thead>
<tr>
<th>recto</th>
<th>verso</th>
</tr>
</thead>
<tbody>
<tr>
<td>BWV 1080/18.1 incipit</td>
<td>BWV 1080/18.1 conclusion</td>
</tr>
<tr>
<td>recto</td>
<td>verso</td>
</tr>
<tr>
<td>BWV 1080/18.2 incipit</td>
<td>BWV 1080/18.2 conclusion</td>
</tr>
</tbody>
</table>

44 The original bifolium status is instead correctly restored in the above mentioned facsimile edited by Sergio Vartolo.
I am inclined to consider Marpurg\textsuperscript{45}, perhaps together with Agricola, responsible for the misunderstanding concerning the Cp XII and Cp XIII and their falsified order. Marpurg, in his *Abhandlung von der Fuge*\textsuperscript{46} mentions the *rectae* and *inversae* versions of the two pieces precisely in the wrong order of the print, and this strikes even more in a context of didactic treatment of the invertible and reversible fugue (*in der Gegenbewegung und Verkehrung*). Besides, also Agricola in his keyboard tablature copy of the KdF\textsuperscript{47} continues the erroneous tradition of inverting the order *rectus/inversus* of the Cp XIII (even if restoring the correct order of the Cp XII): BWV 1080/13.2 becomes *Contrapunto a 3. da rovesciare* [sic] while BWV 1080/13.1 instead the *Contrapunto a 3. roversciato*. The discrepancy with respect to the printed order of BWV 1080/18 is evident and further increases the suspicion that the person (or people) responsible for the *Abklatschvorlage* of the *Fuga a 2 Clav.* was (or were) not the one (or the ones) who prepared those of BWV 1080/12 and, above all, 13, maybe even working at a distance from each other.

Here it is necessary to make some considerations on what many, starting with Marpurg and Agricola, have probably misunderstood: the subject itself of BWV 1080/13. The pair BWV 1080/13 elaborates indeed a subject which is a *diminutio* in violin style and with *Triolen* of the main subject of the KdF. But the matrix – and this, in my opinion, is the origin of the misunderstanding –, i.e. the “*Fuga da roversciare*”, does not start with the *recta* version of the subject as can be seen in Fig. 3A, in the diminished form of Fig. 8, but with the subject inverted by contrary motion (Fig. 3B), in the diminished form of Fig. 7. Therefore the second mirror couple (Cp XIII) begins with the diminished and together inverted by contrary motion version of the KdF main subject. Marpurg and Agricola have probably thought that the matrix was BWV 1080/13.2, since it presented in the very first subject enunciation the diminution of the subject in straight form, which in reality is the product (in the *inversa* version) of the inversion by contrary motion of BWV 1080/13.1 subject, in its turn used in an inverted form! Hence falling into the same mistake reproduced more recently by Dürr\textsuperscript{48}.

Fig. 7 shows how the *diminutio* used as a subject at the beginning of BWV 1080/13.1, which we will see later is the matrix of the pair, relates to the subject by contrary motion of Fig. 3B.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Fig7.png}
\caption{Thema inversum}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Fig7.png}
\caption{Diminutio (BWV 1080/13)}
\end{figure}

\textsuperscript{45} Vartolo (*op. cit.*) hypothesizes that the heirs of Bach might have entrusted Marpurg with the whole final editing and printing process, arguing also his possible influence on the choice of the title “*Die Kunst der Fuge*”.

\textsuperscript{46} Bach Dok III, 655, pp. 48-51.

\textsuperscript{47} Berlin, Staatsbibliothek zu Berlin, Preußicher Kulturbesitz, Am. B. 58.

Moreover, it should be noticed that Bach, not satisfied with a standard fugal answer to this new subject elaboration by contrary motion, uses in the Bass of BWV 1080/13.1 a fugal answer that is, in turn, a contrary motion version of the subject of Fig. 7, corresponding to the *diminutio* of the original subject in straight form (as can be seen in Fig. 3A) as shown in Fig. 8. Therefore the matrix is by itself, following the historical terminology, already a *Fuga contraria* (the answer being the subject by contrary motion), which later, by virtue of the reversion of the polyphonic structure, is called also *Fuga riversa*, hence *Fuga contraria e riversa*.

This corresponds to what occurs in the *incipit* of the Cp V, even if its fugal answer is in stretto form, i.e. enters before the whole subject enunciation ends. Also the parts’ disposition is analogous: the *Altus* enunciates a stepwise version of the main subject by contrary motion in D minor, then the *Bassus* answers with the same stepwise subject but in straight form and always in D minor (Fig. 9).

In our matrix BWV 1080/13.1 it is only the third entry in Soprano 1, logically, to present the initial subject diminished (and so again by contrary motion) in the normal fugal
answer key of the Dominant (Fifth up), i.e. A minor, this time with the standard alteration in the *incipit* (A-D becomes D-A) but with no alteration in the end of the answer: the final A of Soprano 2 in bar 5 remains therefore transposed a Fifth up in Soprano 1 in bar 13, hence an E and not a D as shown instead in Fig. 4B.

If we analyze the behavior of this third entry in the double inversion (by contrary motion and, in accordance with the Multiple Counterpoint, by reversion of the voices) from the upbeat of bar 9 in the *forma inversa* BWV 1080/13.2 (see Fig. 10), we will notice that all what was said regarding BWV 1080/12 occurs also here.

*Fig. 10*

Indeed, this third entry (this time in the *Bassus*) is in G minor and not in A minor, as it should be if this was the *versio recta* from which to generate the mirroring. Therefore the version which has in its exposition an anomalous fugal answer in G minor is always and only the *inversa* one: the matrix, the first one to be composed, is by itself self-sufficient and always duly provided with the answer in the Dominant key. As it was explained before, the lower Fifth key (Subdominant) is a pure consequence of the whole compositional structure inversion, maintaining the melodic intervals without any modifications in the
mirrored versions. Also James Higgs – who analyzed the contrapuntal cycle in the original print order before the Rust’s BG edition and did not know the Autograph P200 – wrote on this matter: “I think where can be little doubt that what appears as the inversion [BWV 1080/12.1 and BWV 1080/13.1 which he found published respectively after BWV 1080/12.2 and BWV 1080/13.2] was really first written”\textsuperscript{49}.

This would be a sufficient reason to conclude the discussion but there are some further considerations which support this analysis. The final part of the matrix BWV 1080/13.1 presents a conclusive Dominant pedal point in the Bassus, therefore in the conventionally most appropriate voice to make use of such a device in proximity of the end of the piece. At the same time Soprano 1 enunciates by the way of epilogue the diminished subject in straight motion: it ends with the first note of bar 70 and follows with a sort of coda on the final Tonic harmony. It is important to observe the natural behavior of the Bassus in the matrix: its ascending scale up to D and the final Octave downward leap with the repetition of the conclusive Tonic (just in the natural foundation voice), the final descending and relaxing motion of the two sopranos between the last triplet and the last chord, where Soprano 1 plays the Fifth A (Fig. 11).

\textbf{Fig. 11}

Everything becomes much less “natural” in BWV 1080/13.2 (Fig. 12): now we have a Tonic (even major) pedal (hence there is not anymore the tension created by the Dominant in the versio recta BWV 1080/13.1) in a middle voice; bars 70 and 71 become now less fluent from the compositional point of view (even if more contrapuntally astonishing!): the elegant motion of the former Bass becomes the middle part with descendent scale crossing the

\textsuperscript{49} J. Higgs, \textit{op. cit.}, p. 64.
new Bassus to emphatically re-ascend later, and - above all - as practically never succeeds in Bach and in all the music of his times, the highest and melodically prominent part, Soprano 1, leaps at the end from the leading note C sharp to the final major Third note, F sharp, instead to the Tonic (in the outermost voices the leading note must in fact be always followed by the Tonic). This behavior is a consequence of the inversion+reversion requirements, and becomes astonishing and musically superlative, since generated by the less intricate musical situation of the recta form BWV 1080/13.1.

As has been said, the mistake of the print edition was then uncritically repeated by all the editions until Rust and, as far as the sequence rectus/inversus of the Cp XIII is concerned, also by numerous critical editions and even recently. Schmieder’s page numbering (unfortunately fallaciously reinterpreted by the new re-edition of his Verzeichnis) remains therefore correct following the order offered by the Autograph P200 and by the Rust edition of the Bach Gesellschaft.

Some important considerations derive from the above presented reasoning and from the diplomatic analysis of the print source. It is likely that the original print titles of BWV 1080/12.1 (Contrapunctus inversus a 4) and of BWV 1080/13.1 (Contrapunctus inversus a 3) were engraved exactly as indicated by Bach in his Abklatschvorlage. In the first place, it is also possible to hypothesize that he didn’t want to give any title to the inversions, considering each of them as a second part of the same Contrapunctus inversus, or using something similar to that “alio modo” which appears in the Fuga a 2 Clav.. Secondly, it

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50 See above, footnote 27.
cannot be excluded that, as in the case of BWV 1080/13.2, also the Abklatschvorlage of the inversa form BWV 1080/12.2 (which in the print has a rather different appearance if compared with BWV 1080/12.1) was prepared not by Bach himself. Therefore the editors, confused, could have interpreted as the inverted forms the matrixes, whose titles included the adjective inversus (as has been said, without understanding that such an adjective was referred to the compositional device, and so it was right be specified it the matrix, and didn’t indicate at all the forma inversa). With no doubt it is their responsibility to have formulated the titles printed in the beginning of BWV 1080/12.2 (Contrapunctus inversus. 12 à 4) and BWV 1080/13.2 (Contrapunctus a 3), almost surely not by Bach. What’s more: the number “12” inserted in the title of BWV 1080/12.2 seems to be, in all likelihood, a later addition (“fast wie nachträglich eingefügt”) to a title probably not formulated by Bach. This clue, the fact that the formae rectae titles of the two mirror pairs are plausibly written by Bach and likely without any by him unequivocally established number, and, furthermore, all the handled issue of the false ordering (definitely contrary to the Bach’s intents) of the rectus/inversus sequence in the Cp XII and XIII, weaken the positions of those who maintain that the edition reflects Bach intents up to the Cp XIII. Butler’s research stated that the page numbers 26-27-28 of the Abklatschvorlage of the Canon per augmentationem in Contrario motu (Beilage 1), once the Canon was repositioned after the other three Canons, bring through a countdown to re-establish the 6 empty pages left for the likely missing mirror fugue (or in any way for a fourteenth Contrapunctus) and to trace a second Lieferung (which, according to some scholars, would constitute in the initial intents a separate second volume) from the Cp XI on. The elements I have exposed above and their implicit consequences seem to partially weaken also these Butler’s conclusions. Had Bach numbered so clearly the pages of the Canon per Augmentationem to assure a precise order of the pieces, then why would he not do the same with the two mirror couples, inserting the page numbering at least in the matrixes to be printed each before the respective inverted form? And why, if we assume that it was really Bach to give the titles appearing in BWV 1080/12.1 and 13.1, didn’t he insert in BWV 1080/13.1 title the number “13”, and “12” in BWV 1080/12.1 (even if one could hypothesize in this case a cancellation made by the editors, and a consequent erroneous insertion of the number “12” into the title of the inverted form BWV 1080/12.2 printed as first)? It seems to me rather illogical and unlikely that Bach – in whatever phase and anyway, according to the general opinion, without having yet completed all the material to be printed – would have numbered already the pages of the Abklatschvorlage of a piece to be inserted as the last one (the Canon

51 Butler, in his above mentioned recent article identifies, following Koprowski’s distinction of the notational style, the engraver of BWV 1080/12.1 with Johann Georg Schübler, while the engraver of BWV 1080/12.2 with his brother Johann Heinrich, see p. 112.

52 Erich Bergel, Bachs letzte Fuge. 1985, p. 36

53 Again Bergel, p. 55: “Wenn […] die Ziffer 12 nicht auf Anweisung Bachs im Titel der vierstimmigen Spiegelfuge erscheint, sondern erst nachträglich von einer anderen Person beigefügt worden ist, dann ergibt sich hieraus […], daß […]alle iegenigen falsch liegen, die sie als Beweis für die angebliche Authentizität der Reihenfolge der Fugen 1-12 in der Erstausgabe betrachten“. Bergel however oversees the fact the whole title of BWV 1080/12.2, and not only the likely subsequent addition of the “12”, does probably not go back to Bach, unlike of the piece mistakenly presented later in the edition BWV 1080/12.1, whose title (without any number?) seems plausibly to be original.


per Augmentationem) without having consistently done the same with the material to be collocated before. Such further critical considerations produce new incentives to the discussion and to the issues, far from being resolved yet, regarding the ordering (at least) from the Cp XII on, as intended by Bach in his last months.

Finally let me mention briefly Rust’s remarks which, in this context, offer a further stimulus. Following Moritz Hauptmann, he considered the Cp XII, preceded by the Cp XIII and the Fuga a 2 Clav. – influenced by compositional-structural reasons and in order to give plausibility to the debated passage of the Nekrolog56 – as the last piece of the cycle, therefore placing the mirror fugues in the end of a work he regarded absolutely complete by itself. It is important to remind that, even if the Rust’s thesis has been questioned and the critics are inclined to see the four Canons as the last group of the variations cycle, in the Autograph it is precisely the mirror fugue couple to conclude the early collection (the following canon looks indeed as a first fair copy of the augmentation canon to be found at pages 32-33, probably written out later after the mirror fugue in three voices).

III

Some clarification on the Canones BWV 1080/16 and 17

The three Canons per Augmentationem in Contrario Motu BWV 1080/14, alla Decima Contrapunto alla Terza BWV 1080/16 and alla Duodecima in Contrapunto alla Quinta BWV 1080/17 make use, in addition to the implicit canonic rules, of the Double Counterpoint resources. To the strict imitation of the dux made at different intervals by the comes, is added then the device to reverse (switch) the parts according to the rules of the Multiple Counterpoint, in this case Double. While in the first quoted canon Bach resorts to the most common Double Counterpoint at the Octave57, in the following cases it is respectively Double Counterpoint at the Third and at the Fifth to be applied.

In the critical and analytical literature on the KdF, as far as it is known to me, a correct explanation has never been given regarding the obligations of these two counterpoint types and their utilization in BWV 1080/16 and 17, always too quickly assimilated by the scholars with the Double Counterpoint at the Tenth and at the Twelfth.

If, on one hand, some of the early counterpoint treatises – as for example, remaining in the Bach circle, the Abhandlung by Marpurg58 – have assimilated the Double Counterpoint at the Third with the one at the Tenth, and the Double Counterpoint at the Fifth with the one at the Twelfth, dealing simply with the ones at the Tenth and at the Twelfth, the precision with which Bach specifies the difference in the titles isn’t at all a fact of no importance, since (as it will be seen more in detail) the composer, following the most orthodox Italian


57 As it has been specified in the first title wanted by Bach for the piece, Canon per augmentationem in Contrapunto all’octava, which, according to what has been reported in the annotation by Johann Christoph Friedrich in the first leaf of the Beilage 1, became subsequently reformulated in the current one during the plate engraving phase, see KB, pp. 49-50.

58 F. W. Marpurg, Abhandlung von der Fuge, 1753, p. 177 and 185.
Renaissance tradition, distinguishes in a clear way the Double Counterpoint at the Third and at the Fifth from the one at the Tenth and at the Twelfth, and uses therefore only the first ones, those indicated in the titles of BWV 1080/16 and 17\textsuperscript{59}. Let’s proceed in an orderly fashion.

BWV 1080/16

The Canon alla Decima \textit{[in]} Contrapunto alla Terza is a canon where the \textit{comes} imitates the \textit{dux} at the Tenth (up), governed by the device of Double Counterpoint at the Third, hence reversible according to the following table, which is on the basis of the aforesaid contrapuntal typology:

\begin{center}
\begin{tabular}{|cccccccccccc|}
\hline
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\
3 & 2 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\hline
\end{tabular}
\end{center}

From this table, in accordance with the traditional rules of the invertible counterpoint, can be deduced both the simultaneous intervals resulting from the reversion (Unison becomes Third after the reversion, Second remains as such, Third becomes Unison, Fourth becomes Second and so on), as well as the typologies of the possible voices reversions. In the latter case, the first and the third cell, both bold typed, indicate the most common and used manners of reversion: it is possible to leave in its place the higher voice (first cell from the left) and transpose the lower voice one Third up, or (third cell from the left) to leave in its place the lower voice and transpose a Third down the higher one.

It is likewise possible, in order to reverse completely the parts disposition, to move the lower voice one Octave up and the higher voice one Tenth down (see the tenth cell, in grey). It is precisely this procedure to be used by Bach in the canon, which is therefore in the Counterpoint at the Third and not at the Tenth as too many critics have written, for, according to the rules of the Double Counterpoint at the Tenth\textsuperscript{60}, if the \textit{comes} of bar 5 is reversed one Tenth down in bar 44, the \textit{dux} of bar 1 should remain in its place, \textit{in loco}, from bar 40 and not be reversed an Octave up, as it occurs in BWV 1080/16.

Therefore: the \textit{dux} \textit{(bassus)} of bar 1 moves an Octave up in bar 40 (becoming \textit{cantus}), the \textit{comes} of bar 5 \textit{(cantus)} moves one Tenth down in bar 44 (becoming \textit{bassus}). Besides, it is important to note that starting from bar 40 the canon, previously at the Tenth, becomes a Canon at the Octave (down), as a consequence of the parts reversion according to the rules of the Double Counterpoint at the Third (in the grey-highlighted possibility: 10 down/8 up).

Scholars have mistakenly and superficially interpreted the reversion of bar 44 as an

\textsuperscript{59} I have suggested some of the following remarks to Sergio Vartolo, who has published them on the pp. 76 e 77 of the introductory essay to the above mentioned facsimile.

\textsuperscript{60} Here follows the table of Double Counterpoint at the Tenth. Highlighted is the case which would have to occur in bar 44 if Double Counterpoint used by Bach had been at the Tenth:

\begin{center}
\begin{tabular}{|cccccccccccc|}
\hline
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
10 & 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\
\hline
\end{tabular}
\end{center}
application of Double Counterpoint at the Tenth, since Soprano moves a Tenth down, without taking into account the Double Counterpoint rules and different reference tables.

**BWV 1080/17**
The Canon alla Duodecima in Contrapunto alla Quinta is a canon where the comes imitates the dux at the Twelfth (up), governed by the device of Double Counterpoint at the Fifth, hence reversible according to the following table, which is on the basis of the aforesaid contrapuntal typology:

From this table, in accordance with the traditional rules of the invertible counterpoint, can be deduced both the simultaneous intervals resulting from the reversion (Unison becomes Fifth after the reversion, Second becomes Fourth, Third remains as such, Fourth becomes Second and so on), as well as the typologies of the possible voices reversions. In the latter case, the first and the fifth cell, both bold typed, indicate the most common and used manners of reversion: it is possible to leave in its place the higher voice (first cell from the left) and transpose the lower voice one Fifth up, or (fifth cell from the left) to leave in its place the lower voice and transpose a Fifth down the higher one.

It is likewise possible, in order to reverse completely the parts disposition, to move the lower voice one Octave up and the higher voice a Twelfth down (see the last cell, in grey). It is precisely this procedure to be used by Bach in the canon, which is therefore in the Counterpoint at the Fifth and not at the Twelth as too many critics have written, for, according to the rules of the Double Counterpoint at the Twelfth, if the comes of bar 9 is reversed one Twelfth down in bar 42, the dux of bar 1 should remain in its place, in loco, from bar 34 and not be reversed an Octave up (here, due to better stylistic arrangement of keyboard writing, a Fifteenth up, i.e. an Octave up + repetition of Octave), as it occurs in BWV 1080/17. Therefore: the dux (bassus) of bar 1 moves an Octave up (+ repetition of Octave, hence a Fifteenth up, due to better stylistic arrangement of keyboard writing and compass) in bar 34 (becoming cantus), the comes of bar 9 (cantus) moves one Twelfth down in bar 42 (becoming bassus). Besides, it is important to note that starting from bar 34 the canon, previously at the Twelth, becomes a Canon at the (lower) Octave (+ its Octave repetition,+

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61 The same table appears moreover in Beethoven’s composition notebooks, in the chapter Vom doppelten Contrapuncte in der Duodecime, where right after having reported the conventional table of Counterpoint at the Twelfth, a table of Double Counterpoint at the Fifth is added. See Ludwig van Beethoven’s Studien im Generalbasse, Contrapuncte und in der Compositions-Lehre aus dessen handschriftlichem Nachlasse gesammelt und herausgegeben von Ignaz Ritter von Seyfred, Wien, 1832 a., 268
62 Here follows the table of Double Counterpoint at the Twelfth. Highlighted is the case which would have to occur in bar 42 if Double Counterpoint used by Bach had been at the Twelth:
hence at the Fifteenth), as a consequence of the parts reversion according to the rules of the Double Counterpoint at the Fifth (in the grey-highlighted possibility: 12 down/8 up).

Also in this case the analysts have mistakenly and superficially interpreted the reversion of bar 42 as an application of Double Counterpoint at the Twelfth, since Soprano moves a Twelfth down, without taking into account the Double Counterpoint rules and different reference tables. Moreover, it is easily comprehensible that Bach’s reversion process could have never been the one of Double Counterpoint at the Twelfth: if we reverse the comes one Twelfth down, following the table of footnote 62 and we leave in its place the dux (following the same table), the canon which previously had an imitative answer at the Twelfth would become absurdly a piece for two Basses with an answer at the Unison. Also a possible addition of Octave replications to one of the two parts, in order to improve the keyboard compass in the writing, would generate harmonic intervals of different nature if compared with those above showed in the table of Double Counterpoint at the Fifth.

In this canon, the addition of an Octave replication (hence generating a reversion at the higher Fifteenth) to the reversion at the higher Octave of the dux in bar 34, as has been said before, is dictated by reasons of keyboard writing style: in order to exploit adequately the highest side of the keyboard and not to fall too much in the lowest side (especially in the right hand passages from bar 50 to 66). The addition of an Octave repetition to the reversion at the higher Octave in bar 34 does not modify the interval qualities of Double Counterpoint at the Fifth, since the typology of the simultaneous consonances after the reversion remains the same, simply with addition of one Octave.

BWV 1080/14

A final look at the proceedings – this time fortunately not misunderstood by the critics – applied by Bach in the Canon per Augmentationem in Contrario Motu, the only other in Double Counterpoint. Here we have a Kreis-Fuge that uses the following compositional devices: the augmentation of the comes in respect to the dux, the contrary motion of the comes in respect to the dux and, indeed, Double Counterpoint at the Tenth. Therefore the reversion makes the lower voice become the higher one, and the higher voice the lower one, starting from bar 53. Also in this case, for the above mentioned reasons of keyboard suitability and aesthetics, the contrapuntal voices disposition respects the most convenient musical compass, by an addition of Octave repetition in the reversion (the notes names of the parts remaining however unchanged after the reversion, according to Counterpoint at the Octave).

The analytical survey of the canons BWV 1080/16 and 17 was in my opinion necessary on one hand to understand better the aesthetic and compositional process by Bach, on the other to reestablish correctly the techniques applied, specified meticulously and precisely by the composer in the titles: Double Counterpoint at the Third and at the Tenth. Even if rare by themselves, they cannot be confused nor in any way assimilated (as it has unfortunately occurred later) with those at the Tenth and at the Twelfth.
A skilled connoisseur, even if reluctant to an only arid and theoretical use of such elaborate proceedings, of the process which has brought from the Renaissance on to the codification
of the complex contrapuntal system, the Kantor wanted to demonstrate one more time in the KdF that even the most rare device – asking for in-depth investigation and reflection, hence for a practical and spiritual Übung – could be moulded under his hands into a musical universe of unsurpassed beauty, after the image and similitude, the most adherent that has ever originated from human being, of the Supreme Idea.